

TUNING OF A LAG-LEAD COMPENSATOR USED WITH A FIRST ORDER PLUS AN INTEGRATOR PROCESS

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ABSTRACT

Lag-lead compensators are well known in automatic control engineering. They have 4 parameters to be adjusted (tuned) for proper operation. The frequency response of the control system or the root locus plot are traditionally used to tune the compensator in a lengthy procedure.

A first order with an integrator process in a unity feedback loop of 67.3 % maximum overshoot and 12 seconds settling time is controlled using a lag-lead compensator (through simulation). The lag-lead compensator is tuned by minimizing the sum of absolute error of the control system using MATLAB.

Four functional constraints are used to control the performance of the lag-lead compensated control system. The result was reducing the process oscillation to 2.438 % overshoot and an 0.648 seconds settling time. The performance of the lag-lead compensated system with the present tuning approach is compared with the classical tuning using the root locus technique. The comparison showed that the present tuning technique is superior.

KEYWORDS: Compensator, Root Locus, Overshoot, Frequency Response, Settling Time and Matlab

INTRODUCTION

Lag-lead compensators can improve the performance of linear control systems through the proper tuning of the compensator parameters. There are two schools in designing lag-lead compensators. One of them uses the frequency response specifications of the compensated control system. The other uses its time response specifications.

In the present work we follow the research school using the time response specifications. Still the subject is interested to automatic control researchers. James, Frederick and Taylor (1987) discussed the application of expert system technique to the design of lead-lag compensators for linear SISO systems [1]. Loh, Cai and Tan (2004) studied the auto-tuning of phase lead-lag compensators using the frequency response of the plant using relays with hysteresis [2]. Chang (2004) used phase-lag and phase-lead compensators to control servo control systems [3]. Wang (2006) developed a non-trial-and-error procedure to design lag-lead compensators based on the idea of Yeung-Wang-Chen's graphical-based non-trial-and-error method for 3-parameters lag-lead compensators and Wang's result on the exact and unique solution for single lag and lead compensator design [4].

Zhang, Liu, Dang, Zhang and Ou (2006) used a lag-lead compensator to control asynchronous linear motors for better performance and application to active mass driver control system for vibration control of civil engineering structures [5]. Panda and Padhy (2007) applied the genetic algorithm optimization technique to design a thyristor controlled series compensator-based controller to enhance the power system stability. They considered two types of controller structure: lead-lag and PID.

They used the ISE and ITAE criterion to optimize the controller parameters [6]. Nassirharand (2008) developed an educational software for designing linear compensators based on the Youla parameterization technique and an exact model matching criterion [7].

Cao, Watkins and O'Brien (2008) studied the use of the graphical user interface (GUS) in designing continuous time compensators using MATLAB. Their work was based on using the root-locus plot and the Bode plot [8]. Wang (2009) provided a new approach for phase-lead/lag compensators to achieve the desired specifications of gain and phase margins, for all-pole stable plants with time-delay [9]. Sawhney (2010) used lag-compensators to improve the response of DC motors using settling time, peak overshoot and steady state error [10]. Li, Sheng and Chen (2010) derived the impulse response of the distributed order lead-lag compensator and used it to compute the distributed order lead-lag compensator in MATLAB [11].

Setiawan (2010) presented the evaluation of closed-loop controller responses of a TiNi SMA gripper using lag, lead and lag-lead compensators. He discussed the tuning of the used compensators [12]. hang and Messner (2011) studied the design of the complex proportional-integral-lead (CPIL) compensator using the root-locus plot. They used 2 design strategies for a double integrator and a second order system with a very lightly damped resonance [13]. Zanasi, Cuoghi and Ntogramatzidis (2011) presented the dynamic structure of a new form of lead-lag compensator with complex zeros and poles. They used Nyquist and Nichols planes in the design of the lead-lag compensator satisfying design specifications on gain margin, phase margin and crossover frequency [14]. Zanasi and Cuoghi (2011) presented 3 different methods for the synthesis of lead-lag compensators meeting the specification of phase margin and gain crossover frequency, plus satisfying an additional specification for robust control [15]. Eslami, Shareef, Mohamed and Khajezadeh (2011) studied the simultaneous coordinated designing of power system stabilizer and static VAR compensator damping controller.

They used a lead-lag compensator structure with gain and wash-out. They used the particle swarm optimization to search for the optimal controller parameters [16]. Guzman and Hagglund (2011) described simple tuning rules for the design of feedforward compensators based on IAE minimization with restriction on the process output overshoot and high-frequency gain of the compensator. They considered lead-lag and lead-lag with delay compensators [17]. Saha, Chatterjee and Dutta (2011) used a lead-lag compensator to control the motion of a snake robot using the frequency domain approach.

The compensator concentrated on controlling the steady-state errors and transient response of the actuators [18]. Nandar (2012) proposed a design of a robust power system stabilizer (PSS) considering less control energy using a genetic algorithm. He used a first order lead-lag compensator as a PSS and showed that with less control energy, the robustness and damping effect was guaranteed [19]. Goswami, Sanyal and Sanyal (2012) studied the control of a hydrofoil ship using a compensator to match the desired control specifications. They used a second order lead-lag compensator of two pole-zero pairs [20]. Ntogramatzidis, Zanasi and Cuoghi (2012) presented a comprehensive range of design techniques for the synthesis of lead, lag and PID compensators. They presented a closed-form formulae for the compensator parameters [21].

PROCESS

The process is a first order process with integrator having the transfer function [22]:

$$G_p(s) = 4 / [s(s + 0.5)] \quad (1)$$

The time response of this process in a unit feedback loop without compensation to a unit step input is shown in Figure 1 as generated by MATLAB:

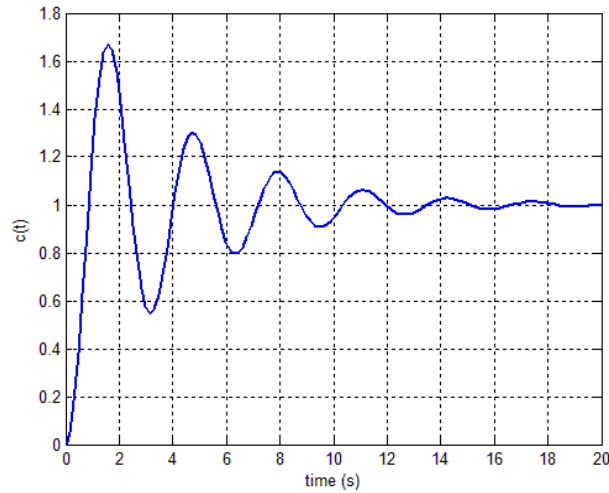


Figure 1: Step Response of the Uncompensated Process in a Unity Feedback Loop

The severity of the output oscillation is measured by its maximum percentage overshoot. It has a maximum overshoot of 67.3 % and about 12 seconds settling time.

LAG-LEAD COMPENSATOR

A lag-lead compensator has 4 parameters [22]:

- First time constant, T_1 .
- Second time constant, T_2 .
- Time constant parameter, b .
- Gain, K_c used in controlling the steady state error of the closed loop control system.

The lag-lead compensator has a transfer function $G_c(s)$ given by [22]:

$$G_c(s) = K_c(1+T_1s)(1+T_2s) / [(1+T_1's)(1+T_2's)] \quad (2)$$

Where: $T_1' = (1/b)T_1$

$$T_2' = bT_2$$

CONTROL SYSTEM TRANSFER FUNCTION

Assuming that the control system is a unit feedback one, its transfer function with $G_c(s)$ and $G_p(s)$ in its forward path is:

$$M(s) = \frac{b_0 s^2 + b_1 s + b_2}{a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4} \quad (3)$$

where: $b_0 = 4K_c T_1 T_2$

$$b_1 = 4K_c(T_1 + T_2)$$

$$b_2 = 4K_c$$

$$a_0 = T_1' T_2'$$

$$a_1 = T_1' + T_2' + 0.5T_1' T_2'$$

$$a_2 = 1 + T_1' + T_2' + 4K_c T_1 T_2$$

$$a_3 = 4K_c(T_1 + T_2)$$

$$a_4 = 4K_c$$

SYSTEM STEP RESPONSE

A unit step response is generated by MATLAB using the numerator and denominator of Eq. 3 providing the system response $c(t)$ as function of time.

LAG-LEAD COMPENSATOR TUNING

The sum of absolute error (IAE) is used as an objective function, F of the optimization process. Thus:

$$F = \int [c(t) - c_{ss}] dt \quad (4)$$

Where:

c_{ss} = steady state response of the system.

The performance of the control system is controlled using three functional constraints:

- **The Maximum Percentage Overshoot, OS_{max} :**

$$C_1 = OS_{max} = 100(c_{max} - c_{ss}) / c_{ss} \quad (5)$$

Which controls the oscillation of the system time response.

- **The Settling Time, T_s :**

$$C_2 = T_s \quad (6)$$

Where: T_s is the time after which the response enters a band of $\pm 5\%$ of c_{ss} and stays within it. This functional constraint controls the speed of response of the system time response.

- **A Stability Constraint**

The control system is a fourth order one which depending on the transfer function parameters may be unstable. Therefore, a constraint or more is required to be imposed to guarantee a stable system during the optimization process. The constraints for the case study in hand are assigned using Routh-Hurwitz criterion of the system characteristic equation:

- Positive compensator parameters.
- Certain function relating the compensator parameters. This function gives the third functional constraint of the optimization process. That is:

$$C_3 = \alpha_1 a_3 - a_1 a_4 \quad (7)$$

Where: $\alpha_1 = (a_1 a_2 - a_0 a_3) / a_1$

C_3 has to be positive for a stable control system.

PARAMETERS LIMITS

- A lower limit of 0.01 is set for the compensator parameters: T_1 , T_2 , b and K_c .

- An upper limit of 10 is set for the compensator parameters T_1 and T_2 .
- An upper limit of 1 is set for the compensator parameters b (this was because with all guessing values of the compensator parameters, b was less than 1).
- An upper limit of 50 is set for the compensator parameter K_c .

TUNING RESULTS

The MATLAB command "fmincon" is used to minimize the optimization objective function given by Eq.4 subjected to the functional inequality constraints given by Eqs. 5, 6 and 7 to provide the lag-lead compensator parameters subjected to the limits mentioned in section 6. The results are as follows:

Controller parameters:

$$T_1 = 9.9995s,$$

$$T_2 = 0.9696s$$

$$b = 0.0385,$$

$$K_c = 24.2083$$

The time response of the compensated system to a unit step input is shown in Figure 2

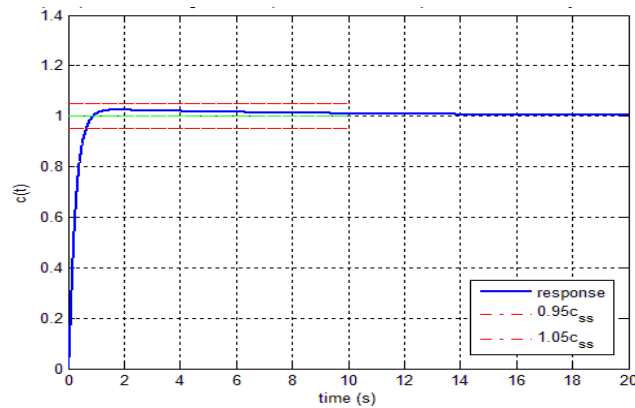


Figure 2: Step Response of the Lag-Lead Compensated System

Characteristics of the control system using the tuned lag-lead compensator:

- Maximum percentage overshoot: 2.438%
- Settling time: 0.648s

COMPARISON WITH ROOT LOCUS BASED COMPENSATOR TUNING

Using the root locus plot technique, Ogata found the lag-lead compensator parameters as [22]:

$$T_1 = 2, \quad T_2 = 10$$

$$b = 10, \quad K_c = 6.25$$

The time response of the system using Ogata compensator tuning is shown in Figure 3

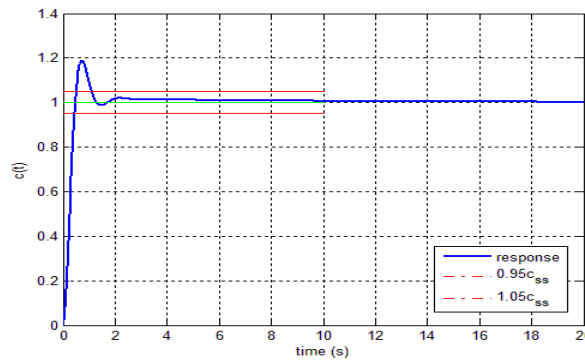


Figure 3: Step Response of the Lag-Lead Compensated Process

Using OGATA Parameters

The haracteristics of the control system using the lag-lead compensator parameters of Ogata are:

- Maximum percentage overshoot: 18.7%
- Settling time: 1.1s

DISCUSSIONS

- The suggested tuning technique og lag-lead compensators is superior compared with techniques based on classical parameter identification such as frequency response or root locus plot techniques.
- Through using a the proposed tuning technique, it was possible to reduce the maximum percentage overshoot of uncompensated process with unit feedback from 67.3% to 2.438% (compared with 18.7% using root locus tuning).
- Using the proposed tuning technique, it was possible to reduce the settling time from about 12 seconds to 0.648 seconds (compared with 1.1 seconds using root locus tuning).
- Even Ogata used the time constant parameter b in the range > 1 , the present study showed that its value is less than one (typically 0.0385).
- The optimization tuning approach used in this work is simple, straight forward, accurate in attaining good control system specifications and provides the compensator parameters in a very small time using MATLAB.

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